**Aircraft Kinematic Tracking with Dynamic Gimbal Camera**

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# Introduction

Tracking another aerial object is a crucial task for many aircraft missions as it provides essential kinematic information to the pilot, flight computer, and base. In the case of human-operated aircraft, visually tracking another aerial object is neither reliable nor accurate enough for analytical purposes. To supply kinematic information to the pilot and base, a gimbal camera system is implemented in various aircraft to deliver useful and precise information.

In this project, an idealized gimbal camera model with two axes of independent rotation is implemented alongside an ASW-28 glider model to report the azimuth and theta angles of another aerial object relative to the ASW-28 FRD and base. The gimbal camera is also equipped with a Lidar sensor to provide range measurements. The model is built in Simulink with a step size of 0.01 seconds and a stop time of 120 seconds.

Through the outputs of the gimbal camera on azimuth, theta, and range, the pilot can quickly determine the location of the other aircraft relative to themselves. Additionally, the onboard flight computer can report the other aerial object’s position and velocity relative to the base.

# Background

## Simulation Initial Conditions

To demonstrate how gimbal measurements return azimuth, theta, and distance, the gimbal-equipped aircraft is positioned directly above the LAX base at an altitude of 1000 meters, while the other aircraft of interest is located to the right of the gimbal aircraft at an altitude of 700 meters (i.e., slightly less longitudinal initial location). Both aircraft utilize a 6-DoF model developed over the semester. An initial velocity of 13 m/s and a body rate of 0 rad/s are set for both aircraft. Consequently, both aircraft will travel towards the North Pole and exhibit phugoid motion.

## Gimbal Definition

The gimbal system in the model is a fixed object attached to the aircraft with two independently rotating axes, y and z, corresponding to pan and tilt angles, respectively. The x-axis of the gimbal is always aligned with the target of interest, representing camera’s center view. Initially, the coordinate system of the gimbal is identical to the aircraft’s FRD coordinate system; however, as the gimbal adjusts its angle, a rotation matrix transformation occurs between the gimbal’s coordinate system and the aircraft’s FRD coordinate system. The figure below illustrates the position of the gimbal relative to the aircraft’s center of mass.

A diagram of a plane

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Figure 1: gimbal camera definition

## Nomenclature

: Definition of gimbal Lidar measurement

: Rate of change of target aircraft’s distance

: Target aircraft (B) with respect to gimbal (G) in gimbal’s frame

: Target aircraft (B) with respect to gimbal (G) in aircraft’s frame (O)

: Target aircraft (B) with respect to gimbal aircraft (O) in gimbal aircraft’s frame (O)

: Rotational matrix from gimbal to aircraft’s frame

: Rotational matrix from aircraft’s frame to NED frame (N)

: Velocity of gimbal aircraft with respect to NED in NED’s frame

: Gimbal tilt angle, measured from x-y plane of gimbal aircraft FRD to target aircraft

: Gimbal pan angle, measure from x-z plane of gimbal aircraft FRD to target aircraft

: Rate of change of and with respect to inertial frame (E)

: Body rate of gimbal aircraft

: Distance of gimbal with respect to reference point on gimbal aircraft in O frame

: Moment of inertia matrix of gimbal camera

# Methods

## Kinematics in Gimbal Aircraft’s Frame

To determine the kinematic information of the target aircraft in the gimbal-equipped aircraft's frame using only data from the gimbal and Lidar measurements, vector addition is employed.

= (1.1)

Equation (1.1) combines the gimbal reading and Lidar measurement to represent the target aircraft’s position relative to the gimbal in the aircraft’s frame. By adding this result to , a constant value denoting the position of the gimbal on the aircraft, a position vector is obtained that allows the pilot to intuitively ascertain the position of the other aircraft. The rotation matrix , consisting of pitch and yaw matrices due to the gimbal’s two-axis rotation, changes as the target aircraft’s position varies. Extracting the angles from the rotation matrix provides and , offering an intuitive representation of the other aircraft’s direction from the pilot’s perspective.

Taking the derivative of Equation (1.1) and incorporating the Coriolis effect yields the velocity of the target aircraft relative to the gimbal-equipped aircraft in the gimbal aircraft’s frame.

(1.2)

## Kinematics in Base’s Frame

The base is designated as LAX using the NED coordinate system. To determine the position vector of the target aircraft using gimbal readings, the equation now involves the addition of three vectors: from the base to the gimbal-equipped aircraft, from the gimbal-equipped aircraft to the gimbal, and from the gimbal to the target aircraft.

= (2.1)

Taking the derivative of equation (2.1) and applying Coriolis equation yields

(2.2)

## Measurement system

With kinematic equations relating to the gimbal, Lidar measurements, and the gimbal-equipped aircraft’s kinematics, the kinematic information of the other aircraft can be calculated. However, these kinematic equations do not account for gimbal dynamics and thus will serve as reference results, because the gimbal possesses inertia and requires time to reach target values depending on the controller design. A measurement system must be developed to supply target values to a dynamic gimbal model. The position of the target aircraft relative to the gimbal in the ECEF frame is presented as follows.

(3.1)

Equation (3.1) performs vector subtraction from the target aircraft's position to the gimbal's position, both in the ECEF frame. The resulting position vector represents the target aircraft relative to the gimbal in the ECEF frame. The representation of this in the gimbal-equipped aircraft’s frame is as follows.

(3.2)

Equation (3.2) is an important component in the construction of the measurement system, as is in cartesian representation. Converting it to spherical coordinates directly yields the azimuth, theta, and R value, and here the R value represents the magnitude of the position vector, analogous to Lidar measurements.

(3.3)

Equation (3.3) converts Cartesian coordinates to spherical coordinates. Here, represents the azimuth angle, which is a z-axis rotation measured from the x-axis, thus is the panning angle for the gimbal camera. However, is measured from the z-axis, as per the definition of spherical coordinates; therefore, is subtracted from ​to derive the angle measured from the x-y plane of the aircraft’s FRD coordinate system, making the tilt angle.

(3.4a)

(3.4b)

## Gimbal Dynamic Model

With and obtained from the measurement system, it is now possible to implement a dynamic gimbal model that uses these values as setpoints. An idealized gimbal camera dynamic model, developed in collaboration with Faheem Chunara, operates under the following assumptions: the moment of inertia matrix J is symmetric, and the gimbal camera lacks a saturation zone. Below is the block diagram of the gimbal camera system.

A diagram of a diagram

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Figure 2: gimbal camera system block diagram

The dynamics block receives external torque as an input to the system and outputs the body rate of the gimbal using the following equation, which is derived from Newton's second law.

(4.1)

Integration of Equation (4.1) yields the actual body rate of the gimbal camera, influenced by the inertia matrix . It should be noted that if is negligible, the true gimbal body rate closely aligns with the desired gimbal rate, which is obtained by taking the time derivative of the desired gimbal angle.

## Gimbal Kinematic Model

The true gimbal rate is passed into the kinematic block, which determines the angular displacement of the gimbal using the quaternion method. The resulting gimbal pan, and tilt angles are used to construct which then is multiplied with , to provide the body rate in the gimbal's frame. Together with from rotational dynamics, can be constructed to help yield results for Equations (1.1), (1.2), (2.1), and (2.2). The Simulink diagram below illustrates the quaternion method in rotational kinematics.

A computer diagram of a computer

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Figure 3: quaternion approach for rotational kinematics

## Discretizing Lidar Distance measurements

To enhance the fidelity of the model, the discretization of the distance measurement is considered. While the gimbal moves towards the target, the Lidar measurement should not return any readings, as there is no physical surface for light wave reflection. To address this in the model, a simple discretizing block is added to the gimbal system. Since the desired gimbal angle originates from the measurement block and the actual gimbal angle is produced via gimbal kinematics, the difference between these angles quantitatively indicates the angular variance until the gimbal aligns with the target aircraft. Therefore, the range measurement R is only permitted to pass values when both the actual pan and tilt angles are within a 5% error margin of the desired pan and tilt angles.A diagram of a machine

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Figure 4: Discretization Block Diagram

## Controller Model

The controller depicted in Figure 1 is responsible for adjusting the input torque to the gimbal system. It compares the actual gimbal angles with the desired gimbal angles and outputs a torque to the gimbal dynamics model. Each axis is controlled by an independent PID controller that provides torque individually, and the gains are adjusted using the Simulink PID tuner.

A diagram of a computer flowchart

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Figure 5: Gimbal PID Controller

# Results

## Desire Vs. True Gimbal Angles

The major distinction between the actual and desired gimbal angles is that the former accounts for the dynamics of the gimbal system, while the latter simply outputs an instantaneous angle of the target aircraft. The plot below illustrates how the desired gimbal system outputs pan and tilt angles, converging to approximately -41.9 and 86 degrees, respectively.A screenshot of a computer

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Figure 6: Desire pan and tilt angle ()

With controller implementation, the true gimbal angle is adjusting to desired gimbal angle, converging to ~ -41.9 in theta and ~ 87 degrees in psi. In the perspective of the pilot, or the ‘O’ frame, the target aircraft is to the right and below.

A screen shot of a computer

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Figure 7: True pan and tilt angles

To further demonstrate the impact of the gimbal dynamic system, the difference between the desired and actual gimbal angles is calculated. Because the control system reaches the target value quickly, the stop time is set at 5 seconds.

A screen shot of a graph

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Figure 8: Difference between desired and true gimbal angles over time

## Kinematic results from True and Referenced Gimbal Angles

Recall that the measurement system outputs the desired gimbal angle, which serves as the set point for the controller. These values are utilized to compute the target aircraft’s kinematic information using the subsequent block, which applies Equations (1.1) through (2.2). It should be noted that because the values from the measurement system are ideal and do not incorporate the dynamics of the gimbal, an additional calculation block is employed to use the actual outputs from the gimbal dynamics to compute Equations (1.1) through (2.2).

A diagram of a machine

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Figure 9: equation (1.1) through (2.2) calculation block

The following four plots are calculated from measurement system, giving the kinematic information of the target aircraft in the frame of gimbal aircraft(O), and the base (N).

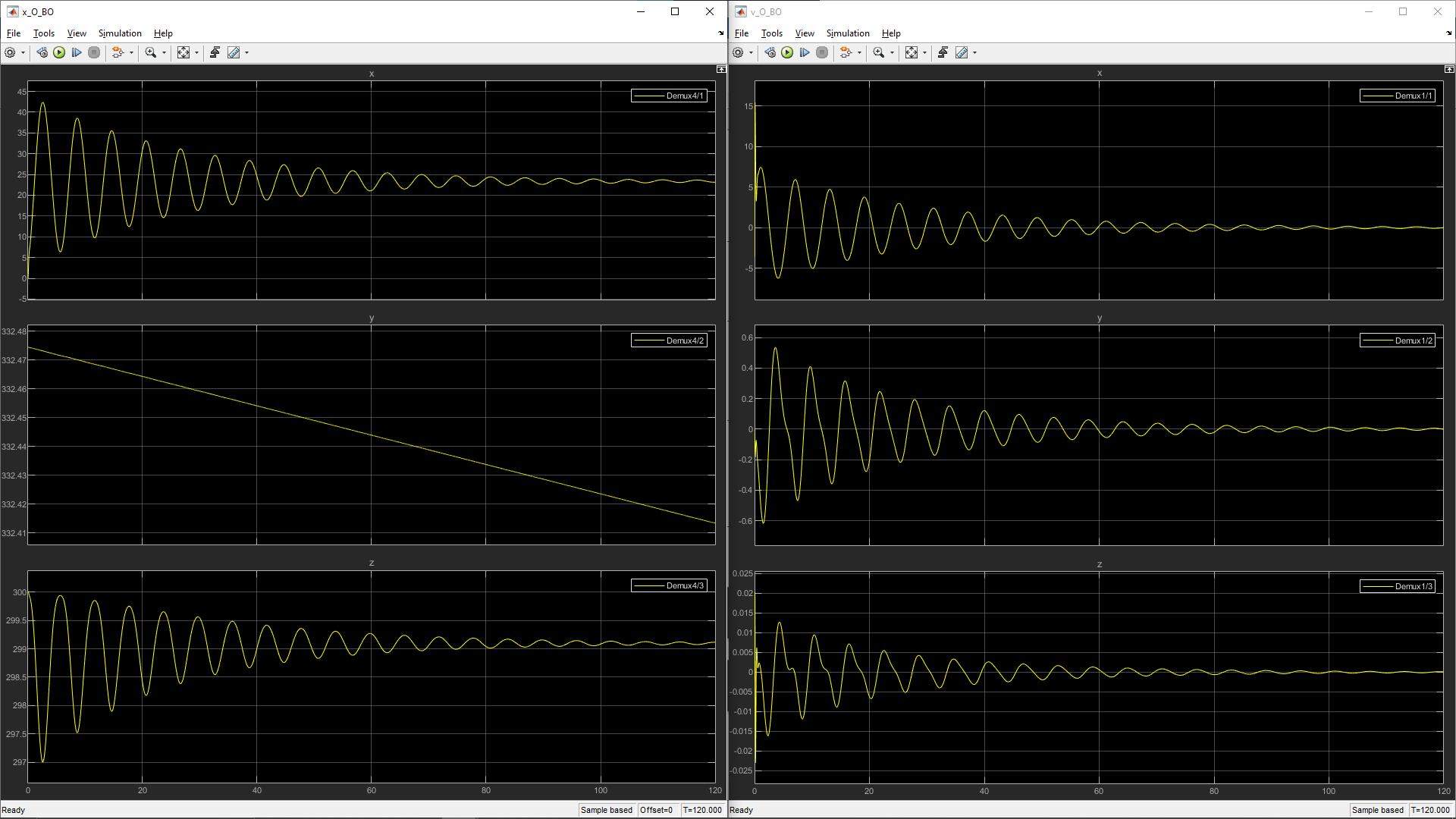


Figure 10: (Left) and (Right)

*A screenshot of a computer

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Figure 11: (Left) and (Right)

The kinematic information of target aircraft accounting for gimbal dynamics, controller, discretization of Lidar produces the following plots for gimbal aircraft (O) and base (N) frame.

A screenshot of a computer

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Figure 12:true (Left) and true (Right)

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Figure 13: true (Left) and true (Right)

The initial cutoff of the signal in the kinematic plots is due to the system's discretization, as the controller occasionally overshoots the set point value, resulting in a loss of information when the gimbal points into empty space. However, as time progresses and both aircraft reach a steady state, the gimbal consistently tracks the target without any signal cutoff.

Overall, the difference between the referenced and actual kinematics becomes negligible as the system stabilizes, although there is significant variance initially when there are no Lidar readings, or when the actual gimbal angle is overshooting or settling. Further tuning of the PID controller would enhance the results and provide more stable readings. In scenarios where the two aircraft are significantly distant from each other, the accuracy demand on the controller increases due to reduced surface reflections for the Lidar. Although the controller's gain is distance-varying, it is kept constant in this simulation for simplicity since the distance between the two aircraft does not vary significantly.

# Conclusion

This project provides an excellent opportunity for our team to become familiar with managing a coupled dynamical system and extracting practical information for further analysis. Throughout the project, I gained experience in deriving kinematic equations that account for two Coriolis effects—those associated with the gimbal and the aircraft's rotation—and in integrating these equations into a block that receives input from the gimbal system. Additionally, a gimbal camera represents a dynamic system with physical properties that must be considered in control scenarios; altering its J-matrix can influence its accuracy and responsiveness. The Lidar sensor on the camera operates by detecting light waves reflected off surfaces; discretizing these detections can enhance the simulation of the kinematics' accuracy.